

VALUATIONS, GAUGES AND APPLICATIONS

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Valuations served to solve many problems in various mathematical fields. Especially, in simple algebra theory, valuations were used to settle important conjectures. Namely, they were used to give examples of simple algebras which are not crossed products, and examples of division algebras with non trivial SK_1 . They were also used to give criterions for some division algebras to have specified subfields such as elementary abelian subfields or Kummer subfields. Many other important constructions were quoted in [JW90], where the authors developed a nice study for division algebras over Henselian valued fields.

We intend in the first part of this course to review many facts on valued division algebras. We will focus essentially on the role of associated graded division algebras to determine necessary and sufficient conditions for a valued division algebra to be tame. This criterion serves in particular to connect the tame part of the Brauer group of a Henselian valued field to the Brauer group of the corresponding graded field (see [B95], [HW99] or [TW15]). We give at the end of this first part a list of the author's contributions in the study of these algebras with some applications on simple algebras.

In the second part of this course, we will introduce the recent theory of value functions and gauges on semisimple algebras. Then we will give some author's results using these 'extended' valuations in the study of simple algebras and involutions of the first kind.

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